THE ISOPERIMETRIC INEQUALITY OUTSIDE CONVEX SETS

The starting point of this course is a classical isoperimetric inequality concerning the perimeter $P(E;H)$ of a set E in a half space H. Namely, given $m>0$, it is well known that among all sets $E\subset H$ of volume m the minimizers of the perimeter of E in H are precisely the half balls of mass m sitting on the hyperplane ∂H. Few years ago, this isoperimetric inequality was extended by Choe, Ghomi and Ritoré in [3] to the case of the relative perimeter of a set E contained in the exterior of a convex set. The aim of this course is to present this general isoperimetric inequality together with a characterization of the equality cases recently obtained in [4].

The course is essentially self contained since most of the preliminary material will be presented in the lectures. However, a basic knowledge of Hausdorff measures, a good background in measure theory and some familiarity with the regularity theory of PDEs is required.

Lecture 1: Basic definitions and properties of sets of finite perimeter, coarea formula, De Giorgi's structure theorem, the euclidean isoperimetric inequality.

Lecture 2: The relative isoperimetric inequality in a half space. First variation of volume and perimeter. The capillarity functional. Young's law.

Lecture 3: Spherically convex sets, Kuratowski convergence, Normal cones and restricted normal cones. Normal bundle and restricted normal bundle.

Lecture 4: The total curvature. An estimate of the total curvature of a set. A Willmore type inequality.

Lecture 5: Λ-minimizers of the perimeter. Density estimates. Regularity of Λ-minimizers.

Lecture 6: Proof of the relative isoperimetric inequality outside a convex set.

Lecture 7: Characterization of the equality cases in the relative inequality outside a convex set.

References

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[4] Fusco, N., Morini M., *Total positive curvature and the equality case in the relative isoperimetric inequality outside convex domains*, Preprint (2021), available at https://cvgmt.sns.it/paper/5054/

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